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ABSENCE OF MASS RENORMALISATION IN SUPERSYMMETRIC QED IN TWO AND THREE DIMENSIONS

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We show that there is no mass renormalisation to one-loop order in supersymmetric QED in $d=2$ and $d=3$, if the value of the bare gauge multiplet mass equals twice that of the matter multiplet. We discuss the possible origin of this effect.

A striking feature of certain supersymmetric field theories in four dimensions is the absence of quantum corrections to parameters of the theory, such as particle masses. Indeed, this property is the reason behind most of the applications of supersymmetry in unification models. In four dimensions this phenomenon of perturbative non-renormalisation is well understood [1]. Non-renormalisation theorems do not hold for $N=1$ supersymmetric models in two and three dimensions. In this letter we point out that for a special choice of the bare mass parameters mass renormalisation is nevertheless absent, at least to one-loop order, in massive supersymmetric quantumelectrodynamics (SQED) in $d=2$ and $d=3$.

In this letter we will concentrate on $SQED_3$. As is well known, in ordinary QED_3 it is possible to have, besides the mass of the charged matter fields, m, a gauge invariant mass for the photon, μ [2,3]. (This mass term can be generalized to the non-abelian case, where it takes the form of a Chern–Simons term. In this letter we will limit ourselves to abelian gauge theories.) Both mass terms allow a supersymmetric extension. The absence of mass renormalisation occurs for the special choice $\mu=-2m$.

$SQED_3$ can be conveniently described by a Majorana superfield $V_\alpha(x,\theta)$, which contains as physcial fields the photon A_μ and a Majorana spinor λ, and by a complex scalar superfield $\Phi(x,\theta)$, with charged physical spin-0 and spin-$\frac{1}{2}$ fields. The action

$$S=\int d^3x\, d^2\theta\, [\tfrac{1}{4}\bar{\nabla}\Phi^*\nabla\Phi+\tfrac{1}{2}m\Phi^*\Phi+\tfrac{1}{4}\bar{V}\not{\partial}(\not{\partial}-\mu)V] , \qquad (1)$$

where we have introduced the gauge covariant derivative

$$\nabla_\alpha\Phi=D_\alpha\Phi+ieV_\alpha\Phi, \quad D_\alpha=\frac{\partial}{\partial\bar{\theta}_\alpha}-(\not{\partial}\theta)_\alpha . \qquad (2)$$

Infinitesimal local gauge transformations are of the form $\delta\Phi=-ie\Lambda\Phi$, $\delta V=D\Lambda$, where $\Lambda(x,\theta)$ is a real scalar superfield. In (1) we have chosen a generalisation of the Feynman gauge. The general form of the V kinetic term reads in an arbitrary covariant gauge with gauge parameters a and a':

$$\tfrac{1}{16}(\bar{F}F-2\mu\bar{V}F-a\bar{D}V\bar{D}D\bar{D}V+2a'\mu\bar{D}V\bar{D}V) , \qquad (3)$$

where $F_\alpha\equiv\bar{D}_\beta D_\alpha V_\beta$ is the gauge invariant field strength. Note that F satisfies the Bianchi identity $\bar{D}F=0$. The choice $a=a'=1$ leads, after some straightforward algebra, to the kinetic term in (1).

The superspace Feynman rules can be derived from (1) and (3). In a general covariant gauge the bare superpropagator of the gauge multiplet takes on the form

$$\Delta_{\alpha\beta}(k;\theta_1,\theta_2)=\frac{\mathrm{i}}{2}\left(\frac{\mathrm{i}\not{k}+\mu}{k^2(k^2+\mu^2)}\,[\bar{\mathrm{D}}\mathrm{D}(k,\theta_1)+2\mathrm{i}\not{k}]\delta_{12}C^{-1}\right.$$

$$\left.-\frac{\mathrm{i}a\not{k}+a'\mu}{k^2(a^2k^2+a'^2\mu^2)}\,[\bar{\mathrm{D}}\mathrm{D}(k,\theta_1)-2\mathrm{i}\not{k}]\delta_{12}C^{-1}\right)_{\alpha\beta} \tag{4}$$

(δ_{12} is the δ-function for the Grassmann-variables θ_1 and θ_2), while the bare matter propagator is

$$\Delta(p;\theta_1,\theta_2)=\mathrm{i}\,\frac{[\bar{\mathrm{D}}\mathrm{D}(p,\theta_1)+2m]\delta_{12}}{p^2+m^2}\,. \tag{5}$$

The three- and four-point interaction vertices are

$$\Gamma^{(3)}_{\alpha}(p,p-k;\theta_1,\theta_2,\theta_3)=-\tfrac{1}{4}\mathrm{i}e[\delta_{13}\bar{\mathrm{D}}_\alpha(p,\theta_1)\delta_{12}-\delta_{12}\bar{\mathrm{D}}_\alpha(k-p,\theta_1)\delta_{13}]\,, \tag{6}$$

$$\Gamma^{(4)}_{\alpha\beta}(p,k;p',k';\theta_1,\theta_2,\theta_3,\theta_4)=\tfrac{1}{2}e^2\delta_{12}\delta_{13}\delta_{14}C_{\alpha\beta}\,. \tag{7}$$

The quantum corrections to the two-point functions can most easily be expressed in terms of corrections to the inverse propagators. From (4) and (5) we find that in lowest order they take the form

$$\Gamma^{(2)}_{\alpha\beta}(k;\theta_1,\theta_2)=\tfrac{1}{8}\{C[\bar{\mathrm{D}}\mathrm{D}(k,\theta_1)+2\mathrm{i}\not{k}]\delta_{12}(\mathrm{i}\not{k}-\mu)$$

$$-C[\bar{\mathrm{D}}\mathrm{D}(k,\theta_1)-2\mathrm{i}\not{k}]\delta_{12}(\mathrm{i}a\not{k}-a'\mu)\}_{\alpha\beta}\,, \tag{8}$$

$$\Gamma^{(2)}(p;\theta_1,\theta_2)=-\tfrac{1}{4}[\bar{\mathrm{D}}\mathrm{D}(p,\theta_1)-2m]\delta_{12}\,. \tag{9}$$

We have calculated the one-loop quantum corrections to (8) and (9), using standard supergraph techniques [4]. The resulting vacuum polarisation, the order-e^2 correction to the inverse gauge superpropagator, reads

$$\Pi_{\alpha\beta}(k;\theta_1,\theta_2)=\tfrac{1}{8}\mathrm{i}e^2\{C[\bar{\mathrm{D}}\mathrm{D}(k,\theta_1)+2\mathrm{i}\not{k}]\delta_{12}(\mathrm{i}\not{k}+2m)\}_{\alpha\beta}I(k^2,m^2,m^2)\,, \tag{10}$$

where the function I is the usual one-loop integral

$$I(p^2,m_1^2,m_2^2)=\int\frac{\mathrm{d}^3q}{(2\pi)^3}\,\frac{1}{[(p-q)^2+m_1^2](q^2+m_2^2)}\,. \tag{11}$$

In the Feynman gauge the one-loop matter self-energy, the order-e^2 correction to the inverse matter propagator, is

$$\Sigma(p;\theta_1,\theta_2)=\tfrac{1}{4}\mathrm{i}e^2[\bar{\mathrm{D}}\mathrm{D}(p,\theta_1)+2(m+\mu)]\delta_{12}I(p^2,m^2,\mu^2)\,. \tag{12}$$

Note that in (10) only the gauge invariant part of the gauge propagator is affected by quantum corrections. Clearly, for $\mu=-2m$ the vacuum polarisation has the same structure as the bare inverse propagator, up to the wavefunction renormalisation. From (12) we see that the same holds for the inverse matter propagator. Thus the choice $\mu=-2m$ implies that there is no one-loop correction to the value of the mass parameters of the theory. In the remainder of this letter we will elaborate on this statement. Note that the result requires supersymmetry: in ordinary massive QED_3 there is a one-loop correction to both masses for any non-zero value of the bare parameters [5]. On the other hand, it has been shown in ref. [6] that in a quite general class of abelian gauge theories in $d=3$, which includes SQED_3, the mass of the photon, μ, is not affected by higher (>1 loop) order quantum corrections. Therefore for $\mu=-2m$ the photon mass in SQED_3, and by supersymmetry the mass of the whole vector multiplet, is not renormalised at all.

The action of SQED_2 [7] in the Feynman gauge takes on the same form as (1), but the gauge transformations now contain $\gamma_5=\gamma_0\gamma_1$: $\delta\Phi=-\mathrm{i}e\Lambda\Phi$, $\delta V=\gamma_5\mathrm{D}\Lambda$. This means that the covariant derivative (2) also contains γ_5. Except for this modification, the perturbative calculation proceeds in exactly the same way as in three dimensions, with the result that now the absence of mass renormalisation holds for $\mu=2m$. The two-dimensional case was treated in more detail in ref. [8]. There we found that also in a non-perturbative approach the choice $\mu = 2m$ leads to considerable simplification of the full propagators.

We have checked explicitly that the absence of mass renormalisation holds in arbitrary covariant gauges. We have also verified that in the Wess–Zumino gauge a component calculation of Feynman diagrams leads to the same one-loop result for the masses.

Our result can be trivially extended to the case where the gauge multiplet is coupled to an arbitrary number N of scalar multiplets, all with mass m. The only modification is that in (10) the vacuum polarisation should be multiplied by a factor N, which affects only the wavefunction renormalisation constant of the gauge multiplet.

At the level of the action (1), the choice $\mu = -2m$ does not appear to give rise to any new symmetries. In particular, there is no additional supersymmetry connecting the gauge and matter multiplets, since this would imply that their masses are equal. Note, that for $\mu = -2m$ the threshold for producing particle–antiparticle pairs coincides with the pole at $k^2 = \mu^2$ in the gauge propagator. This suggests that an explanation of our result might require a description of these models in terms of bound states.

In this letter we have not considered higher order corrections to the matter propagator. The gauge multiplet mass is, in higher orders, protected by gauge invariance [6]. It would be interesting to see, if for $\mu = -2m$, higher order corrections to the mass of the matter multiplet also vanish. Furthermore, it is important to establish whether or not our result extends to the non-abelian analogues of the models considered here.

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